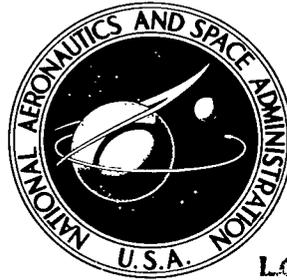


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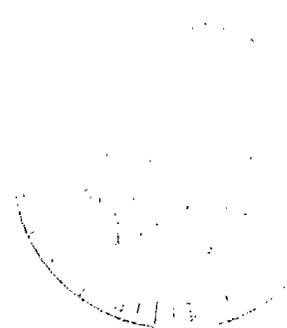


EQUATIONS FOR THE DETERMINATION
OF HUMIDITY FROM DEWPOINT
AND PSYCHROMETRIC DATA

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16. Abstract <p>A general expression based on the Claperon-Clausius differential equation that relates saturation vapor pressure, absolute temperature, and the latent heat of transformation was derived that expresses saturation vapor pressure as a function of absolute temperature. This expression was then used to derive general expressions for vapor pressure, absolute humidity, and relative humidity as functions of either dewpoint and ambient temperature or psychrometric parameters. Constants for all general expressions were then evaluated to give specific expressions in both the International System of Units (SI) and U.S. Customary Units for temperatures above and below freezing. The temperature range considered for all expressions was -50° C to 100° C (-58° F to 212° F) over water and -50° C to 0° C (-58° F to 32° F) over ice. The general expressions, along with the values of the constants that give the specific expressions, appear in a table for easy reference.</p>			
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INTRODUCTION

The measurement of humidity is important to many areas of scientific study. One such area is the reduction and analysis of acoustic data obtained in uncontrolled field environments—aircraft and engine noise data, for example. Acoustic data acquired under these circumstances are usually corrected to a standard day with a specified humidity and temperature. Such corrections are functions of the actual ambient temperature and humidity at the time the data were taken.

There are two primary methods for determining humidity. The first requires that the dewpoint and ambient temperatures be known. The second method requires that the barometric pressure and the psychrometric parameters—dry-bulb (ambient) and wet-bulb temperatures—be known. Both methods normally require reference to tables, the use of special slide rules, or the solution of several complicated equations, all of which are tedious and time consuming and increase the probability of errors in the final result.

The Claperon-Clausius differential equation (ref. 1), which relates saturation vapor pressure, absolute temperature, and the latent heat of transformation, was used as the starting point in the development of the equations presented herein, which give humidity as a direct function of either set of parameters stated above. Humidity can be found with these equations on a computer or calculator with a significant reduction in time and with greater reliability than with the previous methods. Furthermore, the need for tables and special slide rules is virtually eliminated for most practical purposes.

These equations are developed and presented herein in their most general form. The constants for the equations are presented in tabular format for the ambient temperature range from -50°C to 100°C (-58°F to 212°F) and in both the International System of Units (SI) and U.S. Customary Units.

SYMBOLS

A	vapor pressure proportionality factor
a, a_1 , b, b_1 , c, c_1 , d, f, g, k	constants (table 1)
c_i	specific heat of ice at 0° C (32° F), J/kg K (Btu/lbm °F)
c_w	specific heat of water at 15° C (59° F), J/kg K (Btu/lbm °F)
c_{pv}	specific heat of water vapor at constant pressure, J/kg K (Btu/lbm °F)
D	dewpoint temperature, °C (°F)
e_s	saturation vapor pressure, mb (in. Hg)
e_v	vapor pressure, mb (in. Hg)
e_{i0}	vapor pressure over ice at 0° C (32° F), mb (in. Hg)
e_{w0}	vapor pressure over water at 0° C (32° F), mb (in. Hg)
H	absolute humidity, kg/m ³ (lbm/ft ³)
L_i	latent heat of sublimation, J/kg (Btu/lbm)
L_w	latent heat of evaporation, J/kg (Btu/lbm)
m	molecular weight
m_d	molecular weight of dry air, kg mol (lbm mol)
m_v	molecular weight of water vapor, kg mol (lbm mol)
p	atmospheric pressure, mb (in. Hg)
R	specific gas constant, J/kg K (Btu/lbm °F)
R*	universal gas constant, J/kg mol K (Btu/lbm mol °F)



R_d	specific gas constant for dry air, J/kg K (Btu/lbm °F)
R_v	specific gas constant for water vapor, J/kg K (Btu/lbm °F)
T	dry-bulb or ambient temperature, °C (°F) or K (°R)
T'	wet-bulb temperature, °C (°F) or K (°R)
T_o	reference temperature, 0° C (32° F)
U	relative humidity (decimal value)
V	volume, m ³ (ft ³)
ε	ratio of molecular weight of water vapor to molecular weight of dry air, m_v/m_d

Subscripts:

o	reference value
i	ice
w	water
x	dummy variable, replaced by i or w

DEFINITIONS AND RELATIONSHIPS

Vapor Pressure

The vapor pressure of moist air, e_v , is defined as the partial pressure of the water vapor present in the air mass. The vapor pressure is said to be with respect to water (or ice) if the air mass is over a plane surface of water (or ice) at the same temperature and pressure.

Saturation Vapor Pressure

Saturation vapor pressure, e_s , is defined as the vapor pressure that exists when the air mass is at the temperature at which two phases of water coexist in neutral equilibrium.

The saturation vapor pressure is said to be with respect to water (ice) when the air mass is over a plane surface of water (ice) at the same temperature and pressure. Any reduction in temperature results in the formation of dew (frost).

Dewpoint

The dewpoint, D , is the temperature to which moist air must be cooled to become saturated at initial pressure and moisture content. The dewpoint is also the temperature at which the saturation vapor pressure equals the actual vapor pressure. Any further cooling results in the formation of dew or frost. When the dewpoint is at or below freezing, it is called the frostpoint.

Absolute Humidity

Absolute humidity, H , is defined as the mass, m_v , of the water vapor present per unit volume V of air at a given temperature and barometric pressure. Absolute humidity may be expressed in equation form as follows:

$$H = m_v/V \quad (1)$$

To express absolute humidity as a function of the vapor pressure e_v and absolute ambient temperature T , the general law for perfect gases is used to give the following equation:

$$e_v V = m_v R_v T \quad (2)$$

where R_v is the gas constant for water vapor and e_v is with respect to water for T above freezing and with respect to ice for T at or below freezing. Then equation (2) is solved for m_v/V and the result is substituted into equation (1), as follows:

$$H = e_v/R_v T \quad (3)$$

To permit the use of any desired pressure and temperature units, equation (3) may be written in the following, more general, form:

$$H = ke_v/(T + d) \quad (4)$$

where k is the product of all required conversion factors divided by R_v and the value of d determines the temperature units used. Equation (4) is the form used herein. The constants k and d are evaluated in appendix A, and their values are listed in table 1.

Since water vapor is not a perfect gas and is subject to compressibility effects, equations (3) and (4) should contain a compressibility factor in the denominator. However, since the value of that factor is between 1.0000 and 0.9956 for normal atmospheric temperature and pressure ranges (ref. 2), it may be taken as 1.0000 for most purposes with excellent results.

Relative Humidity

The relative humidity of air, U , is defined as the ratio of the partial pressure of the water vapor present at a given temperature and barometric pressure, e_v , to the partial pressure of the water present at saturation for the given temperature and pressure, e_s . In equation form,

$$U = \frac{e_v}{e_s} \quad (5)$$

which gives relative humidity as a decimal value. To express relative humidity as a percentage, the result of equation (5) can be multiplied by 100.

The techniques and procedures recommended in reference 2 require e_v to be evaluated with respect to water for ambient temperatures above freezing and with respect to ice for ambient temperatures at or below freezing and e_s always to be evaluated with respect to water in equation (5).

EXPRESSIONS FOR SATURATION VAPOR PRESSURE

Equation Derivation

When water or ice is transformed into vapor, heat must be absorbed. The total heat absorbed for a water-to-vapor transformation is called the latent heat of evaporation, L_w . The heat absorbed for an ice-to-vapor transformation is called the latent heat of sublimation, L_i . The rate of change in the latent heat of transformation with absolute temperature and at constant pressure may be written as follows:

$$\frac{dL_x}{dT} = c_{pv} - c_x \quad (6)$$

where x is replaced by w for evaporation and by i for sublimation and L_x is the latent heat of evaporation or sublimation, c_{pv} is the specific heat of water vapor at constant pressure, and c_x is the specific heat of water or ice.

Throughout the temperature ranges of interest, the variations in specific heat are small enough so that c_{pv} and c_x may be taken as constant. Hence, for the normal atmospheric temperature range, equation (6) may be integrated as follows:

$$L_x = (c_{pv} - c_x)(T - T_o) + L_{xo} \quad (7)$$

where L_{x0} is the known latent heat of evaporation or sublimation at a reference temperature T_0 .

A form of the differential equation that relates saturation vapor pressure, absolute temperature, and latent heat of transformation is

$$\frac{1}{e_{sx}} \frac{de_{sx}}{dT} = \frac{L_x}{R_v T^2} \quad (8)$$

where e_{sx} is the saturation vapor pressure with respect to water or ice, depending on the ambient temperature. This equation is a form of the Claperon-Clausius equation that relates pressure to temperature in a system in which two phases of a substance are in equilibrium.

The general expression for saturation vapor pressure over water or ice as a function of absolute temperature is obtained by substituting equation (7) into equation (8) and integrating the results, as follows:

$$\log e_{sx} = a \log T + b/T + c \quad (9)$$

where

$$a = (c_{pv} - c_x)/R_v \quad (10)$$

$$b = [(c_{pv} - c_x)T_0 - L_{x0}]/(R_v \ln 10) \quad (11)$$

$$c = \log e_{x0} - a \log T_0 - b/T_0 \quad (12)$$

Equation (9) is derived in reference 1 for saturation vapor pressure over water with e_{sw} in centibars and T in kelvins, as follows:

$$\log e_{sw} = -4.9283 \log T - 2937.4/T + 22.5518 \quad (13)$$

This is known as Magnus' formula.

Equation (9) is the basis for the expressions developed herein for absolute humidity, relative humidity, and vapor pressure. However, the form derived herein is more general than equation (9). That form is obtained by taking the anti-logarithm of equation (9) as follows:

$$e_{sx} = 10^{[c+(b/T)]T^a} \quad (14)$$

and then including a temperature conversion constant, d , to permit temperatures to be expressed in degrees Celsius or Fahrenheit. Thus

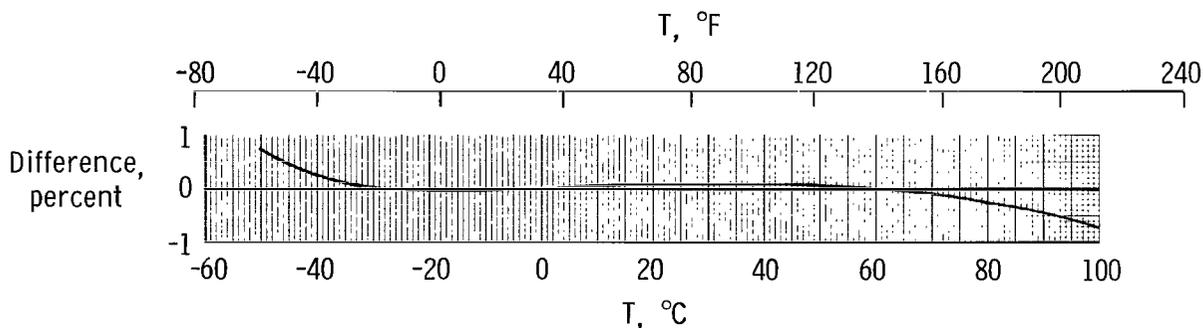
$$e_{sx} = 10^{\{c + [b / (T + d)]\}} (T + d)^a \quad (15)$$

This equation is the desired form for obtaining saturation vapor pressure.

The values of a , b , c , and d in equation (15) depend on the temperature and pressure units desired; on whether saturation vapor pressure is required over water or ice; on the reference temperature used; and on the source used to obtain the values of the other parameters involved in their computation. The values of a , b , c , and d are evaluated in appendix A for the temperature range from -50°C to 100°C (-58°F to 212°F). The results, listed in table 1, give saturation vapor pressure over water and ice for both SI and U.S. Customary Units.

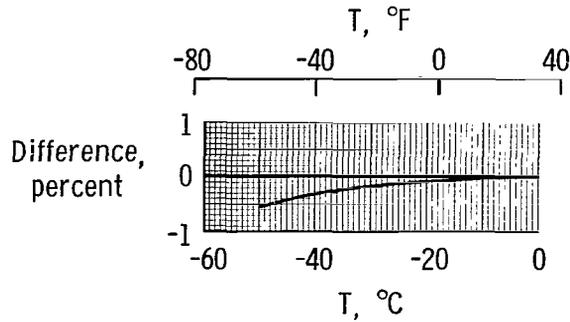
Comparison of Equation Values With Smithsonian Meteorological Tables

Since equation (15) is used to develop the expressions used to find vapor pressure and humidity, any error in it is transmitted to those expressions. Therefore, to test the validity of equation (15), values obtained by using it with the constants given in table 1 for SI were compared to the corresponding values given in the Smithsonian Meteorological Tables (SMT) (ref. 2). The results, expressed as the percentage of difference between the values found with equation (15) and those in the Smithsonian Meteorological Tables, are presented in figures 1(a) and 1(b).



(a) e_{sw} values.

Figure 1. Difference between Celsius values given by equation (15) and those given by Smithsonian Meteorological Tables for e_{sw} and e_{si} .



(b) e_{si} values.

Figure 1. Concluded.

The results of comparing saturation vapor pressure over water, given by

$$e_{sw} = 10^{[23.5518 - 2937.4 / (T + 273)]} (T + 273)^{-4.92830}$$

appear in figure 1(a), and the results over ice, given by

$$e_{si} = 10^{[11.4816 - 2705.2 / (T + 273)]} (T + 273)^{-0.32286}$$

appear in figure 1(b).

Figure 1(a) shows that equation (15) gives values for saturation vapor pressure over water that differ from SMT values by less than 0.1 percent over the temperature range from -30°C to 70°C (-22°F to 158°F) and by less than 0.8 percent over the range from -50°C to 100°C (-58°F to 212°F).

Figure 1(b) shows that equation (15) gives values over ice that differ by less than 0.2 percent over the range from -30°C to 0°C (-22°F to 32°F) and by less than 0.6 percent over the range from -50°C to 0°C (-58°F to 32°F). Since the normal atmospheric temperatures lie within the range from -40°C to 60°C (-40°F to 140°F), equation (15) can be used with the constants in table 1 to find saturation vapor pressure for most practical applications with excellent results, in most cases within the error of the temperature-measuring device used.

EXPRESSIONS FOR VAPOR PRESSURE

Dewpoint Expressions

Because of the definition of dewpoint, equation (15) gives vapor pressure as a direct function of dewpoint (or frostpoint if the ambient temperature is at or below freezing) if T is replaced by D . The same constants in table 1 are used. With the appropriate changes in notation, equation (15) assumes the following form:

$$e_{vx} = 10^{\{c + [b/(D+d)]\}} (D + d)^a \quad (16)$$

where x is replaced by w for ambient temperatures greater than freezing and by i for ambient temperatures at or less than freezing. The units used for dewpoint determine which set of table 1 constants should be used.

Psychrometric Expressions

The most common expression used to find vapor pressure as a function of psychrometric parameters is taken from reference 2 and given here in the functional notation form

$$e_{vx}(T, T', p) = e_{sx}(T') - Ap(T - T') \quad (17)$$

where

$e_{vx}(T, T', p)$	vapor pressure in air over water or ice, depending on T'
T	dry-bulb temperature
T'	wet-bulb temperature
p	barometric pressure
$e_{sx}(T')$	saturation vapor pressure over water or ice, depending on T' (given by eq. (15) evaluated for T')
A	proportionality factor

The pressures e_{vx} , p , and e_{sx} must be in the same system of units.

The proportionality factor A has the form

$$A = (f + gT') \quad (18)$$

where f and g are constants. Although this factor has been determined empirically

and verified by many investigators (refs. 2 and 3), it is credited to Ferrel. The values for f and g appear in table 1; the values for U.S. Customary Units are those obtained by Ferrel, and the values for SI are conversions of those values (app. A). However, reference 2 points out that when the wet bulb is covered by a thin layer of ice, Ferrel's constants are invalid and must be multiplied by 0.882, which is the ratio of the latent heat of evaporation to the latent heat of sublimation. Therefore, for this condition the values given in table 1 for f and g should also be multiplied by this ratio.

A general expression for vapor pressure as a function of psychrometric data is obtained by substituting equation (15) evaluated for T' and equation (18) into equation (17) as follows:

$$e_{vx}(T, T', p) = 10^{[c+b/(T'+d)]} (T' + d)^a - (f + gT')p(T - T') \quad (19)$$

where the values for the constants are given in table 1. The use of over ice or over water values is determined by T' .

EXPRESSIONS FOR HUMIDITY

Once expressions for saturation vapor pressure and vapor pressure have been defined, expressions for absolute and relative humidity can be obtained by substituting equation (15), (16), or (19), evaluated for the appropriate conditions, into the definitions of absolute humidity (eq. (4)) or relative humidity (eq. (5)).

Absolute Humidity as a Function of Dewpoint and Ambient Temperature

The general expression for absolute humidity as a function of dewpoint temperature, D , and ambient temperature, T , is obtained by substituting equation (16) into equation (4) as follows:

$$H(T, D) = k(T + d)^{-1} 10^{[c+b/(D+d)]} (D + d)^a \quad (20)$$

where the appropriate values for the constants are taken from table 1 for specific expressions. The use of over ice or over water values is determined by T .

Absolute Humidity as a Function of Psychrometric Parameters

The general expression for absolute humidity as a function of dry-bulb temperature, T , wet-bulb temperature, T' , and ambient pressure, p , is obtained by substituting equation (19) into equation (4), as follows:

$$H(T, T', p) = k(T + d)^{-1} \left[10^{\{c + [b / (T' + d)]\}} (T' + d)^a - (f + gT')p(T - T') \right] \quad (21)$$

where the appropriate values for the constants are taken from table 1 for specific expressions. The use of over ice or over water values is determined by T' .

Relative Humidity as a Function of Dewpoint and Ambient Temperature

The general expression for relative humidity as a function of dewpoint temperature, D , and ambient temperature, T , is obtained by substituting equation (16) into the numerator and equation (15) into the denominator of equation (5) as follows:

$$U(T, D) = 10^{\left[(c - c_1) + b / (D + d) - b_1 / (T + d) \right]} (D + d)^a (T + d)^{-a_1} \quad (22)$$

To express relative humidity as a percentage, the result of equation (22) can be multiplied by 100. The appropriate values of the constants are taken from table 1. The use of over ice or over water values is determined by D . The subscripted constants differ from the corresponding unsubscripted constants only when the dewpoint temperature, D , is at or below freezing. When D is above freezing, equation (22) reduces to the following expression:

$$U(T, D) = [(D + d) / (T + d)]^a 10^b \left[(D + d)^{-1} - (T + d)^{-1} \right] \quad (23)$$

Relative Humidity as a Function of Psychrometric Parameters

The general expression for relative humidity as a function of dry-bulb temperature, T , wet-bulb temperature, T' , and ambient pressure, p , is obtained by substituting equation (19) into the numerator and equation (15) into the denominator of equation (5) as follows:

$$U(T, T', p) = 10^{-\left\{ c_1 + [b_1 / (T + d)] \right\}} (T + d)^{-a_1} \left[10^{\{c + [b / (T' + d)]\}} (T' + d)^a - (f + gT')p(T - T') \right] \quad (24)$$

where the appropriate values for the constants are taken from table 1. Relative humidity can be expressed as a percentage by multiplying the result of equation (24) by 100. The use of over ice or over water values is determined by T' . The subscripted constants differ from the corresponding unsubscripted constants only when the wet-bulb temperature, T' , is at or below freezing.

APPLYING TABLE 1

All constants appearing in table 1 are evaluated in appendix A. The constants a , b , and c are essentially for the saturation vapor pressure equation, which is fundamental to all the other equations. When saturation vapor pressure over water is specifically required, as in the denominator of the relative humidity equation, the corresponding subscripted constants a_1 , b_1 , and c_1 are used.

The constant d is a temperature conversion constant and depends only on the temperature units required.

The constants f and g are empirical constants that give the proportionality factor essential for vapor pressure as a function of psychrometric data and must be multiplied by 0.882 if the wet bulb is covered by a thin layer of ice.

The constant k is used in the absolute humidity equations to convert from one system of units to another and is a multiple conversion factor divided by the specific gas constant for water vapor.

The use of over ice or over water values for constants is determined by the value of T' or D in those equations in which they appear. When T' or D is less than or equal to freezing, the over ice values are used. When T' or D is greater than freezing, the over water values are used.

Examples using table 1 may be found in appendix B.

SUMMARY OF RESULTS

General equations are derived herein that give saturation vapor pressure, vapor pressure, absolute humidity, and relative humidity as functions of either dewpoint and ambient temperature or psychrometric parameters over a plane surface of either water or ice and for both the International System of Units (SI) and U.S. Customary Units.

The expression for saturation vapor pressure is fundamental to all other expressions, so the values given by it for SI units over water and ice were compared with values for corresponding conditions given by the Smithsonian Meteorological Tables. The comparison showed differences of less than 0.1 percent for the temperature range from -30°C to 70°C (-22°F to 158°F) over water and differences of less than 0.8 percent for the temperature range from -50°C to 100°C (-58°F to 212°F). For over ice, the differences were less than 0.2 percent for temperatures from -30°C to 0°C (-22°F to 32°F) and less than 0.6 percent for the temperature range from -50°C to 0°C (-58°F to 32°F).

*Dryden Flight Research Center
National Aeronautics and Space Administration
June 8, 1976, Edwards, California*

APPENDIX A.—EVALUATION OF TABLE 1 CONSTANTS

The values of the constants a , b , and c for use with SI were found first and converted to their equivalents in U.S. Customary Units. The values of f and g for U.S. Customary Units were taken from references 2 and 3 and converted to their equivalents in SI.

The general expression for saturation vapor pressure derived in the text (eq. (14)) is repeated here for reference:

$$e_{sx} = 10^{(c+b/T)T^a} \quad (14)$$

where a , b , and c are as defined by equations (10) to (12), below:

$$a = (c_{pv} - c_x / R_v) \quad (10)$$

$$b = [(c_{pv} - c_x)T_o - L_{xo}] / (R_v \ln 10) \quad (11)$$

$$c = \log e_{xo} - a \log T_o - b/T_o \quad (12)$$

and T is in absolute temperature units. The units used here for T are kelvins, with the reference temperature, T_o , taken as 273 kelvins.

The values of all other parameters used to evaluate a , b , and c may vary considerably, depending on the source they are taken from. The values used herein, which are listed in table 2, were taken from reference 1 except as noted. They were used because of the close agreement of their results with the values in the Smithsonian Meteorological Tables over the temperature range from -50°C to 100°C (-58°F to 212°F) and not because they were assumed to be the best values available.

Evaluating a , b , and c Over Water

The following evaluation of the constants a , b , and c for equation (14) is the same as that in reference 1 except that e_{sw} is in millibars instead of centibars.

For any specific gas of molecular weight m and gas constant R , it follows from the definition of the universal gas constant R^* and the general law for gases that

$$R^* = mR \quad (25)$$

Therefore, the specific gas constants of water vapor and dry air are related as follows:

APPENDIX A - Continued

$$R^* = m_v R_v = m_d R_d \quad (26)$$

Solving equation (26) for R_v ,

$$R_v = (m_d/m_v)R_d = \frac{1}{\varepsilon}R_d \quad (27)$$

where

$$\varepsilon = m_v/m_d = 18.016/28.97 = 0.622$$

The SI values of a, b, and c can be found by substituting equation (27) into equations (10) and (11) and taking the proper values from table 2, as follows:

$$\begin{aligned} a &= \varepsilon(c_{pv} - c_w)/R_d \\ &= 0.622(1911 - 4185)/287.0 \\ &= -4.9283 \end{aligned}$$

$$\begin{aligned} b &= \varepsilon[(c_{pv} - c_w)T_o - L_{wo}]/(R_d \ln 10) \\ &= 0.622[(1911 - 4185)273 - 2.5 \times 10^6]/(287.0 \ln 10) \\ &= -2937.4 \end{aligned}$$

$$\begin{aligned} c &= \log e_{wo} - a \log T_o - b/T \\ &= \log 6.11 + 4.9283 \log 273 + 2937.37/273 \\ &= 23.5518 \end{aligned}$$

These constants permit saturation vapor pressure to be found in millibars by using equation (14), but only if T is in kelvins. To permit T in degrees Celsius, the approximate temperature conversion

$$T(K) = T(^{\circ}C) + 273 \quad (28)$$

was found to give excellent results. Therefore, by defining a temperature conversion constant d such that d equals zero for T in kelvins and 273 for T in degrees Celsius, equation (14) may be written in the following, more general form:

APPENDIX A - Continued

$$e_{sx} = 10^{[c+b/(T+d)]} (T + d)^a \quad (29)$$

without affecting the values of a, b, and c found above. The values for SI thus found appear in table 1 in the column for over water.

The U.S. Customary Unit equivalents of a, b, c, and d are found as follows. To permit T in degrees Fahrenheit, the approximate conversion

$$T(K) = 5/9 [T(^{\circ}F) + 459.4] \quad (30)$$

was found to give excellent results. If d is made equal to 459.4 for T in degrees Fahrenheit and equation (30) is applied to equation (14), the form given by equation (29) remains valid and the values of b and c change as follows:

$$\begin{aligned} \text{New } b &= (9/5)(\text{Old } b) = (9/5)(-2937.4) \\ &= -5287.32 \end{aligned}$$

$$\begin{aligned} \text{New } c &= (\text{Old } c) + a \log (5/9) \\ &= 23.5518 - 4.9283(\log 5 - \log 9) \\ &= 24.80986 \end{aligned}$$

However, e_{sx} is still in millibars. To convert the millibar value to inches of mercury, the result of equation (29) can be multiplied by 0.02953 by adding the logarithm of 0.02953 to the value of c just found.

$$c = 24.80986 + \log 0.02953 = 23.2801$$

These values of b, c, and d, along with a (which remains unchanged), appear in table 1 for U.S. Customary Units over water.

Evaluating a, b, and c Over Ice

To find a, b, and c over ice for SI, equations (10) to (12) are used with the applicable values from table 2, as follows:

$$\begin{aligned} a &= (c_{pv} - c_i / R_v) \\ &= (1911 - 2060)/461.5 \\ &= -0.32286 \end{aligned}$$

APPENDIX A - Continued

$$\begin{aligned}
 b &= \left[(c_{pv} - c_i)T_o - L_{io} \right] / (R_v \ln 10) \\
 &= \left[(1911 - 2060)273 - 2.834 \times 10^6 \right] / (461.5 \ln 10) \\
 &= -2705.21 \\
 c &= \log e_{io} - a \log T_o - b/T_o \\
 &= \log 6.107 + 0.32286 \log 273 + 2705.21/273 \\
 &= 11.4816
 \end{aligned}$$

These values give e_{si} in millibars for T in kelvins. To permit T in degrees Celsius, let d equal 273, which is equivalent to using equation (28).

The U.S. Customary Unit equivalents of a, b, c, and d for over ice are found in the same manner as for over water.

$$\begin{aligned}
 \text{New } b &= (9/5)(\text{Old } b) = (9/5)(2705.21) \\
 &= -4869.38 \\
 \text{New } c &= (\text{Old } c) + a \log (5/9) + \log (0.02953) \\
 &= 11.4816 - 0.32286 \log (5/9) + \log (0.02953) \\
 &= 10.0343
 \end{aligned}$$

Then, for T in degrees Fahrenheit, if a value of d equal to 459.4, an unchanged value of a, and these values of a, b, and c are substituted into equation (29), the resulting value of e_{si} is in inches of mercury.

The values of the constants a_1 , b_1 , and c_1 in table 1 are the same as the corresponding unsubscripted constants for over water. They are used in the relative humidity equations only to distinguish constants in the denominator from the constants in the numerator.

Evaluating f and g

The constants f and g appear in equations expressing vapor pressure as a function of psychrometric parameters. Equation (17), the general vapor pressure equation, is repeated below for reference:

$$e_{vx}(T, T', p) = e_{sx}(T') - Ap(T - T') \quad (17)$$

APPENDIX A - Continued

where the proportionality factor A has the form

$$A = (f + gT') \quad (18)$$

The expression for A, credited to Ferrel, is given in references 2 and 3 for T' in degrees Fahrenheit as follows:

$$A = 0.000367 \left(1 + \frac{T' - 32}{1571} \right) \quad (31)$$

The values of f and g can be obtained simply by carrying out the operations indicated by equation (18); thus, f equals 3.595×10^{-4} and g equals 2.336×10^{-7} .

To find the Celsius equivalents of f and g, the complete second term of equation (17) is used, with A replaced by equation (18). The term then becomes $(f + gT')p(T - T')$. Since equation (17) is independent of temperature units, this term is as well. Therefore, Fahrenheit temperatures may be assumed. Then, to convert equation (31) to an expression for Celsius temperatures, equations (28) and (30) are combined to give the following conversion

$$T (\text{°F}) = 9/5[T(\text{°C}) + 32] \quad (32)$$

and this expression is substituted into the term $(f + gT')p(T - T')$. This gives $\left[(9/5)f + (9/5)^2gT' + (9/5)(32)g \right]p(T - T')$. Substituting the Fahrenheit values for f and g into this expression gives $(6.60 \times 10^{-4} + 7.57 \times 10^{-7}T')p(T - T')$. Comparing this expression with the term $(f + gT')p(T - T')$, the Celsius equivalent of f is 6.60×10^{-4} and the equivalent of g is 7.57×10^{-7} . The values of f and g are independent of the vapor pressure being measured over water or ice. Therefore, in either system of units, f and g have the same values over both water and ice.

Evaluating k

The constant k appears only in expressions involving absolute humidity. Equation (3), which defines absolute humidity as a function of vapor pressure, is repeated here for reference:

$$H = e_v/R_vT \quad (3)$$

where T is in kelvins. To convert this to an expression for T in degrees Celsius, equation (28) is applied, as follows:

$$H = e_v/\left[R_v(T + 273) \right] \quad (33)$$

The units chosen for H in SI are kilograms per cubic meter, and since e_v is in millibars, H must be converted to newtons per square meter by multiplying

APPENDIX A - Concluded

equation (33) by $100 \text{ N/m}^2 \text{ mb}$, giving the form

$$H = k(T + d)^{-1} e_v \quad (34)$$

where k equals $100/R_v$ and d equals 273 . Substituting the value for R_v from table 2 gives k equal to $100/461.5$, which equals $0.21668 \text{ kg } ^\circ\text{C}/\text{mb m}^3$ over both ice and water.

To find the U.S. Customary Unit equivalent for k , equation (30) is applied to equation (3), which gives H equal to $(9/5)e_v/[R_v(T + 459.4)]$. Temperature is now in degrees Fahrenheit. For e_v in inches of mercury, this expression must be multiplied by $211.405359 \text{ N lbm m/kg (in. Hg) ft}^3$ to convert H to pounds mass per cubic foot. Then equation (35) may be written in the form of equation (34) for U.S. Customary Units, where k equals $(211.405359)(9/5)/461.5$ or $0.82455 \text{ lbm } ^\circ\text{F}/(\text{in. Hg ft}^3)$ over both ice and water. When the wet bulb is covered by a thin layer of ice, the values for f and g in table 1 must be multiplied by 0.882 , which is the ratio of the latent heat of evaporation to the latent heat of sublimation.

The constants for table 1 are now complete.

APPENDIX B.—EXAMPLES USING TABLE 1

Determining the Proper Equation

Table 1 is used to find the proper equation to solve for a desired quantity. The equation that should be used to find a quantity should be chosen according to the type of data available. The forms containing T, T', and p are used for psychrometric data, and the forms containing T and D are used for dewpoint data. Only one form for saturation vapor pressure is given in table 1; it may be used for data of either type.

Once the equation has been chosen, the values for its constants can be chosen according to the temperature range and units of the given data. If T' and D are less than or at freezing, the section entitled Over ice is used for the forms containing them; otherwise the Over water section is used. For the saturation vapor pressure equation, the Over ice section is used for T less than or at freezing and the Over water section is used for T greater than freezing. The column for SI is used for data in SI units, and the U.S. Customary Units column is used for data in U.S. Customary Units.

Example 1

Problem: Given an ambient pressure, p, of 29.7 inches of mercury; a dry-bulb temperature, T, of 75° F; and a wet-bulb temperature, T', of 65.5° F, find relative humidity, U.

Solution: Find the correct equation in table 1 and solve it using the given data. Relative humidity for psychrometric data is required; therefore, the correct equation form is that for U(T, T', p). The values for the constants are taken from the Over water section since T' is above freezing and from the column for U.S. Customary Units since the given data are in U.S. Customary Units. The correct equation is, therefore,

$$U = (T + 459.4)^{4.9283} 10^{[-23.801 + 5287.32 / (T + 459.4)]} \left[(T' + 459.4)^{-4.9283} 10^{[23.2801 - 5287.32 / (T' + 459.4)]} - 10^{-4} (3.595 + 0.002336T') p (T - T') \right]$$

Solving this equation for the given data gives U equal to 0.603 or 60.3 percent.

Example 2

Problem: Given an ambient temperature, T, of 12.2° C and a dewpoint temperature, D, of -10.6° C, find relative humidity, U.

Solution: Relative humidity for dewpoint data is required; therefore, the correct equation form is that for U(T, D) in table 1. The units of the given data are

APPENDIX B - Concluded

in SI and D is below freezing; therefore, the equation constants are taken from the Over ice section and the SI column in table 1. The correct equation is, therefore,

$$U = (T + 273)^{4.9283} (D + 273)^{-0.32286} {}_{10}^{-1} \{-12.0702 - [2705.21 / (D + 273)] + [2937.4 / (T + 273)]\}$$

Solving this equation for the given data gives U equal to 0.173 or 17.3 percent.

Example 3

Problem: Given an ambient temperature, T, of 67.8° F and a dewpoint temperature, D, of 63° F, find absolute humidity, H, and relative humidity, U.

Solution: For absolute humidity, the equation form is that for H(T, D) with the constants taken from the Over water section since D is above freezing and from the column for U.S. Customary Units since the data are in U.S. Customary Units. The correct equation for absolute humidity is, therefore,

$$H = 0.82455 (T + 459.4)^{-1} (D + 459.4)^{-4.9283} {}_{10}^{-1} [23.2801 - 5287.32 / (D + 459.4)]$$

Solving for the given data gives H equal to 9.1×10^{-4} pounds mass per cubic foot.

For relative humidity, the equation form is that for U(T, D); however, since D is above freezing, the values for a, b, and c will be the same as their corresponding subscripted constants a_1 , b_1 , and c_1 ; therefore, the reduced form of U(T, D) may be used. That form is

$$U = [(D - d) / (T + d)]^{a_1} {}_{10}^{-1} b_1 \left[(D + d)^{-1} - (T + d)^{-1} \right]$$

The constants used are the same as for H, above. The correct equation for relative humidity is therefore

$$U = [(D + 459.4) / (T + 459.4)]^{-4.9283} {}_{10}^{-1} - 5287.32 \left[(D + 459.4)^{-1} - (T + 459.4)^{-1} \right]$$

Solving for the given data gives U equal to 0.846 or 84.6 percent. A check shows that the long form for U(T, D) gives identical results.



REFERENCES

1. Holmboe, Jörgen; Forsythe, George E.; and Gustin, William: Dynamic Meteorology. John Wiley and Sons, Inc., 1957.
2. List, Robert J.: Smithsonian Meteorological Tables. Sixth ed. Smithsonian Institution Press, 1971.
3. Marvin, C. F.: Psychrometric Tables for Obtaining the Vapor Pressure, Relative Humidity, and Temperature of the Dew Point. W. B. No. 235, U.S. Government Printing Office, 1941.

TABLE 1.—EQUATIONS AND CONSTANTS FOR FINDING SATURATION VAPOR PRESSURE, VAPOR PRESSURE, ABSOLUTE HUMIDITY, AND RELATIVE HUMIDITY AS FUNCTIONS OF EITHER DEWPOINT OR PSYCHROMETRIC DATA FOR EITHER SI OR U.S. CUSTOMARY UNITS

(a) Equations

$$e_s = (T + d)^{a_1} 10^{\{c + [b / (T + d)]\}}$$

$$e_v(T, D) = (D + d)^{a_1} 10^{\{c + [b / (D + d)]\}}$$

$$e_v(T, T', p) = (T' + d)^{a_1} 10^{\{c + [b / (T' + d)]\}} - (f + gT')p(T - T')$$

$$H(T, D) = k(T + d)^{-1} (D + d)^{a_1} 10^{\{c + [b / (D + d)]\}}$$

$$H(T, T', p) = k(T + d)^{-1} [(T' + d)^{a_1} 10^{\{c + [b / (T' + d)]\}} - (f + gT')p(T - T')]$$

$$U(T, D) = (T + d)^{-a_1} (D + d)^{a_1} 10^{\{[(c - c_1) + b / (D + d) - b_1 / (T + d)]\}}$$

$$U(T, T', p) = 10^{-\{c_1 + [b_1 / (T + d)]\}} (T + d)^{-a_1} [10^{\{c + [b / (T' + d)]\}} (T' + d)^a - (f + gT')p(T - T')]$$

TABLE 1.—Continued

(b) Constants

	Over water ¹		Over ice ²	
	SI	U.S. Customary Units	SI	U.S. Customary Units
a	-4.9283	-4.9283	-0.32286	-0.32286
b	-2937.4	-5287.32	-2705.21	-4869.38
c	23.5518	23.2801	11.4816	10.0343
d	273	459.4	273	459.4
³ f	6.600 × 10 ⁻⁴	3.595 × 10 ⁻⁴	6.600 × 10 ⁻⁴	3.595 × 10 ⁻⁴
³ g	7.570 × 10 ⁻⁷	2.336 × 10 ⁻⁷	7.570 × 10 ⁻⁷	2.336 × 10 ⁻⁷
k	0.21668	0.82455	0.21668	0.82455
a ₁	-4.9283	-4.9283	-4.9283	-4.9283
b ₁	-2937.4	-5287.32	-2937.4	-5287.32
c ₁	23.5518	23.2801	23.5518	23.2801

¹Use Over water section for forms containing T' or D above freezing.

²Use Over ice section for forms containing T' or D at or below freezing.

³Constants must be multiplied by 0.882 if wet bulb is covered by thin layer of ice.

TABLE 1.—Concluded

(c) Definitions

D	dewpoint if above freezing, frostpoint if at or below freezing, °C (°F)
e_s	saturation vapor pressure, mb (in. Hg)
e_v	vapor pressure, mb (in. Hg)
H	absolute humidity, kg/m^3 (lbm/ft^3)
p	ambient pressure, mb (in. Hg)
T	ambient or dry-bulb temperature, °C (°F)
T'	wet-bulb temperature, °C (°F)
U	relative humidity, decimal value; may be expressed as a percentage by multiplying by 100

TABLE 2.—PARAMETERS AND VALUES USED FOR EVALUATING
TABLE 1 CONSTANTS FOR SI

Specific heat of ice at 0° C (32° F), c_i , J/kg K	2060
Specific heat of water at 15° C (15° F), c_w , J/kg K	4185
Specific heat of water vapor at constant pressure, c_{pv} , J/kg K	1911
^a Vapor pressure of ice at 0° C (32° F), e_{i0} , mb	6.107
Vapor pressure of water at 0° C (32° F), e_{w0} , mb	6.11
Latent heat of sublimation at 0° C (32° F), L_{i0} , J/kg	2.834×10^6
Latent heat of evaporation at 0° C (32° F), L_{w0} , J/kg	2.5×10^6
Molecular weight of dry air, m_d , kg mol	28.97
Molecular weight of water vapor, m_v , kg mol	18.016
Universal gas constant, R, J/kg mol K	8313.6
Specific gas constant for dry air, R_d , J/kg K	287.0
Specific gas constant for water vapor, R_v , J/kg K	461.5

^aValue taken from reference 2.



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